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NUMERICAL ANALYSIS OF THE RX450 FREE-MOTION FLIGHT

By

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Presented to the Faculty of Engineering and Life Sciences In Partial Fulfilment Of the Requirements for the Degree of

SARJANA TEKNIK

In

AVIATION ENGINEERING

FACULTY OF ENGINEERING AND LIFE SCIENCES

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APPROVAL PAGE

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I hereby declare that this submission is my own work and to the best of my knowledge, it contains no material previously published or written by another person, nor material which to a substantial extent has been accepted for the award of any other degree or diploma at any educational institution, except where due acknowledgement is made in the thesis.

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ABSTRACT

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Rocket technology advancement is one of a few attempt that are aimed to reduced the operating and manufacturing costs of a rocket launch. Lembaga Penerbangan dan Antariksa Nasional(Lapan) are currently in the early stages of developing rocket, LAPAN's programs that will bring LAPAN engineers to have the ability to design and manufacture orbital rockets is in progress. The RX series rocket is a sounding rocket that is used for various experimental flight circuits to test various technologies that can help advance Indonesia rocket technology. A Rocket must follow a designated trajectory or an orbital flight path in order to deliver the payload into the correct orbit to achieve its mission successfully. So in order to design a suitable controller, it is important to obtain the free-motion flight which is a flight trajectory of the rocket. The free motion flight another are then compared to another flight that has an influenced of a small perturbation, the result of this comparison will show the differences of motion or attitude of the rocket that will determine the conclusion of this thesis.

Keyword: Rocket, Free-Motion, Small Perturbation

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List of Abbreviations

RCS	Raction Control System
ISA	International Standard Atmosphere
COESA	Committee On Extension to the Standard Atmosphere
WGS	World Geodetic System
ESC	Electronic Speed Control

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Dedicated to my parents

CHAPTER 1 INTRODUCTION

1.1 Background

Rocket technology advancement is one of a few attempt that are aimed to reduced the operating and manufacturing costs of a rocket launch but at the same time it also has to add new capabilities and payload assisted by the latest technological innovations. The attempt to reduce launch costs has been tried before by various innovations such as National Aeronautics and Space Administration (Nasa) attempt to reuse the space shuttles and the boosters for the shuttle program, Indian Space Research Organization (ISRO) by keeping the payload simple and very light such as the mission to Mars where the total cost of launching is 10 times cheaper than the NASA Mars mission (Mahajan, 2014). these missions can be achieved none other than because of various developments in manufacturing, materials, computing and others technology. This efficiency and cost reducing effort is very important because often times spacecraft development projects for both rockets and satellites are often far exceeding the predetermined budget and exceeding the deadline that create even bigger cost overruns. as an example of NASA's Space shuttle project which have an estimated cost of 20 million dollars or 130 million USD (Inflation adjusted) for each launch ended up at about 450 million USD (Heppenheimer, 1999) and James Webb space telescope that have projected cost from 1 billion dollars to 9 billion USD with a planned launch originally for 2007 and just being launch at the end of 2021. Figure 1.1 shows all the 3 vehicle

So it is not a surprise to say that Lembaga Penerbangan dan Antariksa Nasional(Lapan) that are currently in the early stages of developing rocket technology have to take some lesson from those project. Currently LAPAN's programs that will bring LAPAN engineers to have the ability to design and manufacture orbital rockets is in progress. The program called Roket Pengorbit Satelit(RPS), a series

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FIGURE 1.1: Space Shuttle (Left), ISRO Mars, JWST (Right)

of Sounding rocket called RX-series. The RX series rocket is a sounding rocket that is used for various experimental flight circuits to test various technologies that can help advance Indonesia rocket technology. One of the technologies that can help advance rocket technology is rocket control technology.

A Rocket must follow a designated trajectory or an orbital flight path in order to deliver the payload into the correct orbit to achieve its mission successfully. To follow a trajectory, Rocket usually has a guidance system that consists of GPS, sensors, radar, onboard computer, and communication system to provide stability and maneuver for the rocket. The differences between sub-orbital flight to an orbital flight and traditional atmospheric flight are shown in the Figure 1.2, most of the sub-orbital flight are barely passing the 100 Km altitude. (Goehlich, 2002)

Rocket atmospheric flight are notorious for its wide range of uncertainties. this uncertainties is causes by the High-speed winds and turbulence blowing into the rocket. So it is important to obtain and analyze the free-motion flight which is a rocket flight without any influence from the control system from the rocket.

The Free-motion flight will consist of a undisturbed motion meaning that there will be no random external forces other than the atmospheric pressure and disturbed free motion flight, in which there will be some external forces applied during the flight at a certain flight phase to duplicate a small perturbation.

Both of the free motion flight another are then compared and analyze based on

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FIGURE 1.2: Sub-Orbital Flight Illustration

their information of the performance thru out the entire flight. The study will analyze the rocket performance such as the Range, Endurance, Attitude (Pitch,Roll,Yaw), Velocity and etc. The result of this comparison will show the differences of motion or attitude of the rocket that will determine the conclusion of this thesis.

1.1.1 Rocket Dynamics

True dynamics of rocket is naturally nonlinear, especially during the part of atmospheric flight due to turbulence, clouds, winds. These conditions will deviate the rockets from its natural trajectory. The deviation from its natural motion can cause several affect in the later part of the flight, but before we get into that there are 3 forces acting during the flight i.e Thrust, Gravity, Aerodynamic. Aerodynamic forces acting only during the atmospheric flight or where the air pressure density is starting to increase. To control the rocket there are several types of flight control system during different phase of the flight. Flaps and grid fins only have influenced during the atmospheric flight while RCS(Reaction Control System) can provide control during both flight regime. On the software side there is apply control technique such as gain scheduling and Proportional Integral Derivative (PID) loops can be struggling due to the unpredictability of the nonlinear effect at these situation.

1.1.2 Lapan Rocket Program

Lapan is currently building a series of RX (Rocket Experimental) sounding rocket, these rocket are classified by the tube diameter that are ranging from 100 mm up to 550 mm. Fig. 1.3 gives various rockets that LAPAN has been developed. Currently Lapan only tested in a Single-stage solid fueled rocket configuration, Lapan planned is to certified each of this single-stage rocket that are then will be combined together. This combined or stacked up versions of these single stages is called Roket Pengorbit Satelit (RPS). The Programs objective stated by Lapan is that these serial rocket flight test and manufacture is part of the program to train LAPAN Pustekroket engineers to design and manufacture multistage rockets. Below is the data of Lapan RX-series variant and apogee or the highest point of the flight that each rocket reached:

- RX-100 (10 Km);
- RX-250 (70 Km);
- RX-320 (40 Km);
- RX-420 (45 Km);
- RX-450 (84 Km);
- RX-550 (N/A).

1.1.3 Basic information of the rocket

RX450 rocket is an experimental single-stage sounding rocket built in-house by LA-PAN. It has Solid-fueled with Hydroxyl Terminated Polybutadiene (HPTB). Flight control surface (FCS) consist of 4 movable forward fins. Recent experimental flight test proved that the rocket capable of reaching 100 km of altitude. Programs objective: To train LAPAN Pustekroket engineers to design and manufacture multistage rockets.

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FIGURE 1.3: Lapan RX-series Rocket

The Figure 1.3 below is showing all the Lapan RX series rocket. all of this rocket are part of the RPS program. each of these rocket will be assembled together to create a customize satellite launcher vehicle.

Lapan RX-450 Technical Specification. (Dito, Haryadi, et al., 2019) (Jamaludin, 2010) (Sumaraw, 2011) (Nuryanto et al., 2010)

The Figure 1.4 shown the major part of the RX-450 rocket.

1.2 Problem Statement

The results of this thesis are certainly expected to provide assistance to the reader as an additional consideration for the type of rocket control that will be used in for a similar rocket design. Therefore, The main problems discussed in this thesis are:

- How the rocket move when unperturbed?
- How the rocket move when experiencing small perturbations?

Part	Specification
Motor Diameter	$450 \mathrm{mm}$
Motor Mass	834 kg
Structure Mass	$186.5 \ { m kg}$
Rocket Head	370 kg
Total Mass	1201 kg
Total Impulse	1,523
Burn Time	$16 \sec$
Average Thruust	95 kN

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TABLE 1.1: RX-450 Technical Specification

240 s

• Is the performance as expected?

1.3 Research Objectives

The objectives of this research are accordingly then to:

- Derive the equations of motion for the rocket;
- Build nonlinear numerical simulation model for the rocket;

Specific Impulse

- Simulate the unperturbed motion;
- Simulate the response of the rocket under small perturbation;
- Verify the expected range and altitude of the rocket.

1.4 Research Scope and Limitation

Simulation Model The simulation will consist of a results that is built based on several software for building the physical modeling, simulation trajectory and environment, and control system. Physical Modelling Missile-Datcom will create the database for the rocket dynamics. SolidEdge create the 3D CAD modeling to help create the several reference data for the CG and moment of inertia as a comparison

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FIGURE 1.4: RX-450 Major Parts

to the data provided by Lapan (Gunawan, Djatmiko, Wicaksono, & Abrizal, 2020). A cad drawing is presented at Figure 3.4

Simulink will simulate the rocket flight this include the environment, the control system, rocket forces (Thrust, aero, etc) and trajectory that consist of several trajectory such as the free flight (undisturbed), free flight (disturbed) and controlled flight, This thesis will not include the atmospheric friction that can cause heating to the rocket during all the phases of the flight.

The assumption is that the rocket are behave differently between 2 of the different flight condition and the result can be use as a consideration of to what type of flight controller needed to be use that might be suitable for this type of rocket.

In this thesis, the assumptions taken are:

- The rocket geometry and inertia are of RX-450;
- WGS84 is adopted for the Gravity acceleration model;
- The rocket is rigid.

1.5 Significance of the Study

The results of this research are expected:

- This thesis can be use for further development for Lapan's RX450 rocket control design.
- This thesis can provide a basic understanding of how RX450 motion during the flight.
- This thesis can provide a basic understanding of how RX450 respond to a perturbation that effect the motion of the rocket during the flight.

CHAPTER 2 LITERATURE REVIEW

2.1 Free Body Diagram

3 Figures below are the representative of the free body diagram illustration for all 3 axis in Longitudinal FBD Figure 2.1, Lateral FBD Figure 2.2 and Directional FBD Figure 2.3.

2.2 Equations of Motion

To use the equation of motion on a non-rigid, variable mass system. we define all mass confined by the outside wall as S_r , and the exhaust area as A_e (Cornelisse, Schoyer, & Wakker, 1979).

2.2.1 Reference Frame

Various reference frames is needed to describe rocket orientation and position from deriving the equations of motion. Depending on the mission the reference frame for most rockets that orbit relatively close to the planet or have a relatively short flight length is tied to the earth-bound only reference.

Inertial Reference Frame. For rockets that within a closerăproximity of the earth, the non-rotating geocentric equatorial can be used as an inertial frame. the unit vectors can be defined by this 3×1 matrix.

$$E = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}$$
(2.1)



FIGURE 2.2: RX-450 Lateral FBD





FIGURE 2.3: RX-450 Directional FBD

Rotating geocentric reference frame is used if the rocket flight is relatively long, so the earth rotation is included in the reference frame.

$$E_g = \begin{bmatrix} e_{x_g} \\ e_{y_g} \\ e_{z_g} \end{bmatrix}$$
(2.2)

Vehicle centered horizontal reference frame describes The rocket's orientation and velocity vector in relation to the earth's surface. The origin and center of mass are coincides. The reference frame's orientation is constitutes to a local horizontal system.

$$E_{v} = \begin{bmatrix} e_{x_{v}} \\ e_{y_{v}} \\ e_{z_{v}} \end{bmatrix}$$
(2.3)

Vehicle reference frame origin is from center of mass, x-axis along rocket longitudinal. y and z axis in two principal axis of inertia. The usage of a vehicle reference frame is beneficial because it represent the aerodynamic forces and moments, as well as apparent forces and moments.

$$E_r = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}$$
(2.4)

Relative orientation of the various reference frame

Non rotating geocentric equatorial frame to rotating geocentric frame. a rotating geocentric frame revolves around the z-axis of a non rotating frame at angular velocity. the relative of this orientation is at x- and the x_{g-} axis. $t = t_0$ at Greenwich, the angle H_g at any time and relation between the unit vectors in both system:

$$H_g = \omega_e(t - t_0). \tag{2.5}$$

$$E_g = A_g \times E. \tag{2.6}$$

$$A_g = \begin{bmatrix} \cos H_G & \sin H_G & 0 \\ -\sin H_G & \cos H_G & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(2.7)

Revolving geocentric frame to vehicle centered horizontal frame. The vehicle centered horizontal is acquired from a rotating geocentric frame by two translation and rotations. to acquire the same orientation as the vehicle centered, revolving geocentric frame at Z_g axis over an angle A. Y- axis over angle -(pi/2 + omg).

Translational moves bring the reference frame to the center of mass of the rocket.

$$E_v = A_{vg} \times E_g \tag{2.8}$$

$$A_{vg} = \begin{bmatrix} -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ -\sin\lambda & \cos\lambda & 0 \\ -\cos\phi\cos\lambda & -\cos\phi\sin\lambda & -\sin\phi \end{bmatrix}$$
(2.9)

Vehicle centered horizontal frame to Vehicle frame. Relative orientation is determined by 3 angles called Euler angle. first rotate is Z_o axis over angle, then Y_o axis over angle theta, last X_o axis about angle. The limitation is that if one the angle pitch into 90°, the other two angle is undetermined.

Matrix Arv

$$A_{rv} = \begin{bmatrix} C\theta C\psi & C\theta S\psi & -S\theta \\ -C\varphi S\psi + S\varphi S\theta C\psi & C\varphi C\psi + S\varphi S\theta S\psi & S\varphi C\theta \\ S\varphi S\psi + C\varphi S\theta C\psi & S\varphi C\psi + C\varphi S\theta S\psi & C\varphi C\theta \end{bmatrix}$$
(2.10)

Non rotating geocentric equatorial frame to vehicle frame.

The method of direction cosines is used to fix Euler angles problem. Let A be the transform matrix that transforms the vectors of the XYZ (in capital) frame into vectors of the xyz Frame.

$$E_r = A_r \times E. \tag{2.11}$$

$$E_{r} = \begin{bmatrix} e_{x} \cdot e_{x} & e_{x} \cdot e_{y} & e_{x} \cdot e_{z} \\ e_{y} \cdot e_{x} & e_{y} \cdot e_{y} & e_{y} \cdot e_{z} \\ e_{z} \cdot e_{x} & e_{z} \cdot e_{y} & e_{z} \cdot e_{z} \end{bmatrix}$$
(2.12)

The transformation of A_r can be obtained successively applying the transformation A_g, A_{v_g} , and A_{r_v} .

$$A_r = A_{r_v} \times A_{v_q} \times A_g \tag{2.13}$$

2.2.2 The Dynamical Equation

The position vector of the mass element dM relative to the rocket's mass center is r. External and apparent forces and moments are the terms on the right hand side of these equations.

$$F_c = 2\Omega \times \int_M \frac{\delta_r}{\delta_t} dM, \qquad (2.14)$$

$$F_{r_{e_l}} = -\int_M \frac{\delta_r^2}{\delta t^2} dM, \qquad (2.15)$$

$$M_c = -2\int_M r \times \left(\Omega \times \frac{\delta_r}{\delta_t}\right) dM,\tag{2.16}$$

$$M_{r_{e_l}} = -\int_M r \times \frac{\delta_r^2}{\delta t^2} dM, \qquad (2.17)$$

The relative velocity and acceleration of combustion products with regard to the vehicle's center of mass are represented by the dr/dt and d2r/dt2. The velocity and acceleration of the combustion products relative to the stiff rocket structure are V and a, respectively. the following relation hold:

$$\frac{\delta_r}{\delta_t} = V - u_{cm},\tag{2.18}$$

$$\frac{\delta_r^2}{\delta t^2} = a - a_{cm} \tag{2.19}$$

 u_cm and a_cm are velocity and acceleration of the center of mass relative to the rocket structure.

The Apparent Moments

The Coriolis moment

$$M_{c} = -\frac{\delta I}{\delta_{t}} \times \Omega - mr_{e} \times \left(\Omega \times r_{e}\right) - \int_{A_{e}} \nu \times \left(\Omega \times \nu\right) \left(\rho V.\eta\right) dA_{e} - \Omega \times \int_{M} r \times \frac{\delta_{r}}{\delta_{t}} dM.$$
(2.20)

Complete expression of the Coriolis moment. the integral A_e can be neglected. The angular momentum of the rocket owing to relative motion with respect to the center of mass is represented by the volume integral. This angular momentum is exclusively contributed by moving gasses. since the overall mass of combustion products inside the rocket is quite tiny, and the mean gas flow is approximately parallel to the longitudinal axis This amount of angular momentum is insignificant.

$$M_c = -\frac{\delta I}{\delta_t} \times \Omega - mr_{ex} (\Omega \times r_e)$$
(2.21)

Using eq 2-5 the relative moment can be written

$$M_{rel} = -\frac{\delta}{\delta_t} \int_M r \times \frac{\delta_r}{\delta_t} dM - \int_{A_e} r \times \frac{\delta_r}{\delta_t} (\rho V.n) dA_e.$$
(2.22)

Relative moment using the definition of re.

$$M_{rel} = -\frac{\delta}{\delta_t} \int_M r \times \frac{\delta_r}{\delta_t} dM - \int_{A_e} r \times \frac{\delta_r}{\delta_t} (\rho V.n) dA_e + mr_{ex}.$$
 (2.23)

The surface Ae, the velocity V written as

$$V = V_e + \eta. \tag{2.24}$$

By ignoring the component holding the volume integral in this expression, Re becomes larger than V, and n, as well as the relative velocity, Ucm, of the center of mass, becomes smaller than the mean exhaust velocity, the relative moment can be approximated extremely effectively by.

$$M_{r_{el}} = -mr_e \times V_e \tag{2.25}$$

The Inertial Moment

To elaborate the inertial moment using Eq. 2.15 and Eq. 2.16

The External Forces

In free flight or without thrust, external forces are separated into gravitational forces, pressure forces, and frictional forces.

gravitational forces caused by celestial bodies, resulting in a gravitational field of g at the rocket's location. $W = M_g$, other celestial body can be neglected because the rocket is significantly closer to the earth. field strength is a function of its location vector. R_{cm} of cm of vehicle to center of earth.

Pressure forces dS is a control surface element S, and p represents external forces at dS followed by external at dS, n represents the outward unit normal on dS. The total external force on the rocket, for every closed surface S, then pa

constant pressure for every closed surface S, surface s consisting of surface S_r and exit A_e .

$$F_{p} = -\int_{S_{R}} (p - p_{a}) n dS - \int_{A_{e}} (p - p_{a}) n dA_{e}.$$
 (2.26)

This term simply means that the pressure force is solely related to the rocket's speed relative to the surrounding atmosphere. The total aerodynamic force is the sum of the aerodynamic pressure force and frictional forces, and it is called as aerodynamic pressure force.

$$F_a = -\int_{S_R} \left(p - p_a\right) n dS + F_f \tag{2.27}$$

The sum of the aerodynamic force, gravitational force, and pressure thrust equals the total external force.

$$F_s = F_a + W - \int_{A_e} \left(p - p_a \right) n dA_e \tag{2.28}$$

The External Moments

The location of the center of gravity and the center of mass are differ, however due to the small size of the rocket in comparison to the distance of the attraction center and the distance between center of mass (CM) the difference between the two will be negligible. As a result, the moment caused by gravitational forces is insignificant in comparison to the other moments and can be ignored.

The total moment due to pressure at the closed surface is separated into two parts: the pressure moment and the moment of the pressure force on the surface element dS. The frictional moment and total external moment make up the total aerodynamic moment.

$$M_p = -\int_{S_R} (p - p_a) r \times n dS - \int_{A_e} (p - p_a) r \times n dA_e$$
(2.29)

Total aerodynamic moment with mf is the frictional moment

$$M_a = M_f - \int_{S_R} (p - p_a) r \times n dS$$
(2.30)

Total external moment

$$M_{cm} = M_a - \int_{A_e} \left(p - p_a \right) r \times n dA_e \tag{2.31}$$

The apparent and external forces and moments, as well as inertial moments, were derived using the Equation of Motion.

$$M\frac{dV_cm}{dt} = -2m\Omega \times r_e - mV_e - \int_{A_E} (p - p_a)ndA_e + W + F_a, \qquad (2.32)$$

The Equation of Motion After derived the apparent and external forces and moments as well as inertial moments.

If the there is a thrust misalignment issue and create thrust misalignment moment, and as the impulse thrust is much larger than the pressure thrust, the moment approximated by

$$M_F = r_e \times F. \tag{2.33}$$

The Coriolis force can be ignored since the center of mass flow due to rotation is relatively modest in contrast to the mean exhaust velocity. The Coriolis moment, on the other hand, is incomparable to the aerodynamic and thrust misalignment moments in terms of size. The Coriolis moment is made up of two terms. Because of the exhaust jet and the damping moment, the term "exhaust jet" was coined. Often referred to as the jet dampening moment. This term in the Coriolis moment must be considered because it can reduce damping by 30%.

Eq of motion

$$M\frac{dV_{cm}}{dt} = F + W + F_a \tag{2.34}$$

The thrust F is given by

$$F = -\int_{A_e} V(pV.n) + (p - p_a)ndA_e$$
(2.35)

Vehicle reference frame

$$\frac{dV_{cm}}{dt} = \frac{\delta V_{cm}}{\delta t} + \Omega + V_{cm} \tag{2.36}$$

$$\frac{d}{dt}(I.\Omega) = \frac{\delta I}{\delta t} \cdot \Omega + I \cdot \frac{d\Omega}{dt} + \Omega \times (I.\Omega)$$
(2.37)

Resolve the vectors occurring in eq . 2.36 in the following way:

$$V_{cm} = \begin{bmatrix} u, v, w \end{bmatrix} E_r \tag{2.38}$$

$$\Omega = [p, q, r] E_r \tag{2.39}$$

$$F = \begin{bmatrix} F_x, F_y, F_z \end{bmatrix} E_r \tag{2.40}$$

$$F_a = \begin{bmatrix} X_a, Y_a, Z_a \end{bmatrix} E_r \tag{2.41}$$

$$M_a = \begin{bmatrix} L, M, N \end{bmatrix} E_r \tag{2.42}$$

$$r_e = \begin{bmatrix} x_e, y_e, z_e \end{bmatrix} E_r \tag{2.43}$$

$$g = \left[g_x, g_y, g_z\right] E_r \tag{2.44}$$

 F_y and F_z are relatively small in comparison to the F_x and $Y_e Z_e$ are small compared to X_e , because these phrases are the result of ass, which then will be kept to a minimum. therefore the second-order terms in these numbers are neglected. Substitution of eqs. 2.37, 2.38 into 2.34 leads to

$$M\frac{du}{dt} = M(vr - wq) + F_x + Mg_x + X_a$$
(2.45)

$$M\frac{d\nu}{dt} = M(wp - ur) + F_y + Mg_y + Y_a$$
(2.46)

$$M\frac{dw}{dt} = M(uq - vp) + F_z + Mg_z + Z_a$$
(2.47)

$$I_{xx} = \frac{dp}{dt} = -p\frac{dI_{xx}}{dt} + rq(I_{yy} - I_{zz}) + mx_e(y_{eq} + z_{er}) + L$$
(2.48)

$$I_{yy} = \frac{dq}{dt} = -q\frac{dI_{yy}}{dt} + pr(I_{zz} - I_{xx}) - mqx_e^2 - x_eF_z + z_eF_x + M$$
(2.49)

$$I_{zz} = \frac{dr}{dt} = -r\frac{dI_{xx}}{dt} + pq(I_{xx} - I_{yy}) - mrx_e^2 - x_eF_y + y_eF_x + N$$
(2.50)

2.2.3 The Kinematic Equations

The position and direction of the rocket affects the dynamical equation, thrust, gravitational field of strength, as well as aerodynamic forces and moments. It is necessary to develop an equation that can connect position and orientation to rotational velocity and translational velocity. These called the kinematic equation.

Rocket orientation is determined by the rotation matrix A_r ,

$$\frac{dE_r}{dt} = \frac{dA_r}{dt}E + A_r\frac{dE}{dt}.$$
(2.51)

The unit vectors along the axes of the inertial system are element of the matrix E. Because the vehicle reference system rotates at Ω velocity relative to the inertial system,

$$\frac{dE}{dt} = \frac{\delta E_r}{\delta t} + \Omega x E_r \tag{2.52}$$

The direction cosines between the vehicle reference system and the inertial system are given by this matrix equation, which yields nine first order differential equations in the elements of A_r

$$\frac{dA_r}{dt} = \begin{bmatrix} 0 & \mathbf{r} & -\mathbf{q} \\ -\mathbf{r} & 0 & \mathbf{p} \\ \mathbf{q} & -\mathbf{p} & 0 \end{bmatrix}$$
(2.53)

$$R_{cm} = \begin{bmatrix} X, Y, Z \end{bmatrix} E \tag{2.54}$$

By differentiating we can obtain

$$\frac{dR_{cm}}{dt} = V_{cm} = \frac{d}{dt} [X, Y, Z] E = [u, v, w] E_{rr}$$
(2.55)

Differential equation relate the position of the rocket into its velocity component u, v, andw and direction cosines a_{ij}

$$\frac{d}{dt}[X,Y,Z] = [u,v,w]A_r \tag{2.56}$$

The Position of The Rocket

The position of the rocket relative to the spinning geocentric frame can be computed if the coordinates of the rocket's center of mass in the inertial frame are known.

$$[X_g, Y_g, Z_g]E_g = [X, Y, Z]E.$$
(2.57)

$$\left[X_g, Y_g, Z_g\right] = \left[X, Y, Z\right] A_g^T.$$
(2.58)

 R_{cm} distance from center of earth to the rocket, O and A geocentric latitude and geographic longitude.

$$R_{cm} = \sqrt{X_g^2 + Y_g^2 + Z_g^2} \tag{2.59}$$

$$\sin \Phi = \frac{Z_g}{R_{cm}}, -90^\circ < \Phi < 90^\circ$$
 (2.60)

$$\sin \Lambda = \frac{Y_g}{\sqrt{X_g^2 + Y_g^2}}, \frac{Y_g}{\sqrt{X_g^2 + Y_g^2}}$$
(2.61)

$$\cos \Lambda = \frac{X_g}{\sqrt{X_g^2 + Y_g^2}}, \frac{Y_g}{\sqrt{X_g^2 + Y_g^2}}$$
(2.62)

If the rocket starting position is given by polar coordinates R_{cm}, O, A the starting values of X, Y, Z follow from

$$[X, Y, Z] = R_{cm} [\cos \Phi \cos \Lambda, \cos \Phi \sin \Lambda, \sin \Phi] A_g$$
(2.63)

If geocentric latitude and geographic longitude are known to the orientation relative the vehicle centered frame, express in pitch, yaw and bank angle can be calculated by.

$$A_{rv} = A_r \times A_g^T \times A_{vg}^T.$$
(2.64)

The Orientation of The Rocket

The Solution of the equation of motion yields A_r and the coordinates X, YandZ, let cij be the elements of matrix A_{rv} . then we find for the pitch angle

$$\sin \theta = -C_{13} \tag{2.65}$$

And if $c_{13} = 1$ for the bank angle and yaw angle.

$$\sin\psi = \frac{C_{12}}{\cos\theta}, \sin\psi = \frac{C_{11}}{\cos\theta}$$
(2.66)

$$\sin \phi = \frac{C_{23}}{\cos \theta}, \sin \phi = \frac{C_{11}}{\cos \theta} \tag{2.67}$$

If $c_{13} = 1$ in this case vertical flight, the yaw and bank angle are undetermined.

The Velocity Components in The Vehicle-Centered Horizontal Frame

The magnitude of velocity V_{cm} , flight path angle y, and flight path azimuth u can all be used to describe the rocket translational velocity.

$$V_{cm} = V_{cm} \big[\cos \gamma \cos \psi, \cos \gamma \sin \psi, -\sin \gamma \big] E_v$$
(2.68)

$$V_{cm} = \begin{bmatrix} u, v, w \end{bmatrix} A_{rv} E_v \tag{2.69}$$

Thus the magnitude of the velocity, flight path angle and flight path azimuth can be determined from

$$V_{cm} = \left[\cos\gamma\cos\psi, \cos\gamma\sin\psi, -\sin\gamma\right] = \left[u, v, w\right] A_{rv}$$
(2.70)

If the beginning values are known, the matrix A_{rv} can also be used to determine the initial values u,v, and w.

The rocket angular velocity can be described in terms of the pitch, yaw, and bank rates, which are the times rate of change of the respective angles. Differentiating can be used to determine these rates.

$$\frac{dA_{rv}}{dt} = \frac{dA_r}{dt} A_g^T A_{vg}^T + A_r \frac{dA_g^T}{dt} A_{vg}^T + A_r A_g^T \frac{dA_{vg}^T}{dt}.$$
 (2.71)

Final Equations

2.2.4 Equations of Motion

The Apparent Moments

$$M_{rel} = -mr_e \times V_e \tag{2.72}$$

Total External Force

$$F_s = F_a + W - \int_{A_e} \left(p - p_a \right) n dA_e \tag{2.73}$$

External Moments

$$M_{cm} = M_a - \int_{A_e} \left(p - p_a \right) r \times n dA_e \tag{2.74}$$

The Equation of Motion after derived

$$M\frac{dV_{cm}}{dt} = -2m\Omega \times r_e - mV_e - \int_{A_e} (p - p_a)ndA_e + W + F_a, \qquad (2.75)$$

2.2.5 Gravitaitional Law

Newton's gravitational law implicates.

$$g = g_0 \times (r/ha)^2 = g_0 \times (r/r + hg)^2$$
(2.76)

g is variable for different heights.

The World Geodetic System (WGS) is a cartographic, geodetic, and satellite navigation system that includes GPS. The WGS84 Simulink block model is limited

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FIGURE 2.4: WGS gravity model

for low geodetic height, which is from sea level to about 100 km which is still exceeding the Apogee of the flight. The simulated rocket is expected to reach about 100 km of altitude. thus why WGS84 is used for this simulation.

As shown in Figure 2.4 the gravity force pulling the rocket are remain constant thru out the flight, significant changes happens only during the powered flight for about 16 seconds.

2.2.6 Altitude Measurement

Altitude measurement in a rocket or an aircraft is measured by the hydrostatic equation.

$$\delta P = -\rho.g.\delta \tag{2.77}$$

P is Pressure, ρ is the air density, g is the gravity acceleration, and h is the height. due to its difficulty to measure the gravity acceleration for different altitude, therefore airplane usually measure geopotential altitude (gravity acceleration = constant).

2.2.7 Geopotential Height (h)

Earth's gravity has Altitude and Latitude dependence. Geopotential Height allows us a new reference (or coordinate) to accommodate these changes in gravity.

Thus it can be considered a "gravity-adjusted height". Geopotential height is the adjustment of geometric height (elevation above mean sea level) utilizing the fluctuation of gravity with latitude and elevation is known as geopotential height.

$$\delta(h) = \int_0^h g(\theta, Z) \delta Z \tag{2.78}$$

Thus it can be considered a "gravity-adjusted height".

2.2.8 Geometric Height (h)

Geometric altitude (h_g) Measured the height from mean sea level for its altitude. Meanwhile geometric altitude using the newton gravitational law to equation to earn the gravitational acceleration.

This method however less preferable if it use for low altitude (0 km - 20 km) flight because it's insignificant, thus it often neglected (gravitational acceleration).

CHAPTER 3 RESEARCH METHODOLOGY

3.1 Overview

This thesis research methodology is based on data earned by Simulation modeling, simulation modeling involves a process of designing a model of a real world scenario, that then conducting an experiment with different conditions for further data gathering as a simulation result. The simulation result will then use for further understanding of the control system performance and as an evaluating alternative for Lapan rocket development.

The overview of the research methodology for this thesis is given in Figure 3.1. The figure shows that the physical modeling is created by 3 different software that will be integrated into the Simulink to create the high fidelity simulation. The analysis will be provided in the chapter 3.3.2



FIGURE 3.1: Research Methodology Chart

3.2 Derivation of Equations of Motion

After applying the Newton's law of motion in translation and rotation, and then transformed into body coordinate system, one obtained,

The Apparent Moments,

$$M_{r_{e_l}} = -mr_e \times V_e \tag{3.1}$$

Total External Force,

$$F_s = F_a + W - \int_{A_e} \left(p - p_a \right) n dA_e \tag{3.2}$$

External Moments,

$$M_{c_m} = M_a - \int_{A_e} \left(p - p_a \right) r \times n dA_e \tag{3.3}$$

The Equation of Motion after derived,

$$M\frac{dV_{c_m}}{dt} = -2m\Omega \times r_e - mV_e - \int_{A_e} (p - p_a)ndA_e + W + F_a, \qquad (3.4)$$

3.3 Environments Modelling

The Atmospheric modeling for this simulation is the Committee on Extension to the Standard Atmosphere (COESA). The rocket are expected to achieve 100 Km in altitude during the unperturbed flight thus why it is important to have the right number for all of the atmosphere variable. The 1976 COESA values consist of standard lower atmospheric values for absolute temperature (T), pressure (P), density(ρ), and speed of sound(a), with the input of a geopotential altitude (H).

3.3.1 COESA for Rocket

An orbital or suborbital rocket have a significantly different flight profile compare to regular aircraft. The altitude that a rocket reach compared to an aircraft does change the gravity acceleration, Up to the point where the earth loses it influence to the rocket. COESA simulate this temperature, pressure, density, acceleration caused by gravity, from the data they gather from rocket/satellite flight.

Although the gravitational acceleration differences is very small between CO-ESA and ISA in 100 km altitude. The main reason that rocket use COESA is that the measurement also assumed the temperature distribution at those higher altitude. And COESA allow the calculation above 86 km (geometric) altitude, which is the maximum for the ISA model.

3.4 COESA

3.4.1 COESA

After its establishment in 1953 U.S. Committee on Extension to the Standard Atmosphere (COESA) create 4 versions of the U.S. Standard Atmosphere, 1958, 1962, 1966, and 1976. The COESA model block in Matlab/Simulink implements the most recent versions, the mathematical representation of the 1976 atmosphere model. 1976 COESA values compromise of standard value of the lower atmospheric for absolute pressure, density, temperature, and speed of sound, with geopotential altitude as the input.

The atmospheric densities and temperatures are shown from sea level to 1000 km based on rocket and satellite data and ideal gas theory. The US Standard Atmosphere is equivalent to the International Civil Aviation Organization's Standard Atmosphere below 32 km (ICAO). The function extrapolates data below the geopotential altitude of 0 m (0 feet) and above the geopotential altitude of 85 000 m (about 279 000 ft). It extrapolates temperature and pressure readings linearly and logarithmically (Minzner, 1976, 1977).

An orbital or suborbital rocket have a significantly different flight profile compare to regular aircraft. The altitude that a rocket reach compared to an aircraft does change the gravity acceleration, Up to the point where the earth loses it influence to the rocket. COESA simulate this temperature, pressure, density, acceleration caused by gravity, from the data they gather from rocket/satellite flight.



FIGURE 3.2: COESA Atmospheric model

Although the gravitational acceleration differences is very small between COESA and ISA in 100 km altitude.

The main reason that rocket use COESA is that the measurement also assumed the temperature distribution at those higher altitude and COESA allow the calculation above 86 km (geometric) altitude, which is the maximum for the ISA model

The Figure 3.2 show the atmospheric variable condition such as the temperature, air density, air pressure and speed of sound that are measure from sea level up to 100 km of altitude. From the air pressure data we can see the air pressure are becoming negligible right around 40 km of altitude, this will be an important finding later on in this thesis.

3.5 Gravity Model

The Gravitational force modeling is using the World Geodetic System/WGS84 Gravity model. As I mentioned the rocket only reach 100 Km of altitude, thus why the most significant gravity pull will only come from the earth. Other celestial body gravity influence are too insignificant for this type of mission. The gravity pull also simulated in all 3-axis. The Figure 3.3 show the gravity acceleration of th RX450 in all 3 axis during the undisturbed flight simulation.



FIGURE 3.3: Gravitational Acceleration of RX450

3.6 Physical Modeling

Rocket basic information and performance data is provided by LAPAN which consist of the technical dimension, technical specification, thrust profile, weight profile, and several other. For the vehicle dynamics database, several software is used to obtain the data. Missile-Datcom is use to estimate the vehicle aerodynamics coefficients data. SolidEdge is use to obtain the vehicle mass moment of inertia and also create a physical modeling for figures and illustrations for this thesis. Simulink is use to run the simulation model of the entire rocket flight profile and simulate the NDI control system. SolidEdge software is use to create a CAD 3D model for illustrations and free body diagram.

The Figure 3.4 below show the rocket in 3 different view as a general standard for a technical drawing, these illustration are the result of the 3D CAD modeling to give flexible visual representation tool for this thesis.



FIGURE 3.4: RX450 CAD model

3.6.1 Missile Datcom

Datcom is an aerodynamic design tool that has predictive accuracy for missiles, rocket and aircraft preliminary design. Missile-Datcom is a specialized version only for missiles with fins configuration. Missile-Datcom general purpose is to provide a quick and economic way for estimation of aerodynamic for a wide variety of preliminary design of a missile. This will help the designer to create predictions for wide variety of database for the aerodynamic of each of their missile configuration.

The for005.dat is dedicated for user input file. below is the minimum amount of input to run the program (Blake, 1998). The figure shows the input requirement for the input file to work it consist of the items explained below and can be adjust to what the designer requirement for their rocket. The output file will be the data of each coefficients for each angle of attack that are chosen and also separated into a single case for each mach number that are chosen.

• Starting from namelist FLTCON to define the flight condition for the missile starting from number of alpha (NALPHA) and the alpha number (ALPHA),

number of mach (NMACH) and the mach number (MACH), altitude (ALT) and few other variable.

- REFQ is to define the reference quantities, this allow the user to input the geometry of the vehicle such as : reference area (SREF),longitudinal reference length(LREF), C.G. position (XCG), surface roughness height(ROUGH).
- AXIBOD which define the shape and the symmetric of the body, e.g type of nose (TNOSE), center body length (LCENTR), nozzle diameter (DEXIT).
- FINSET define the number and shape of the fin, and FINSETn means you can added more than 1 set of fin going aft.

The missile datcom input file for the flight condition are created according to the possible flight attitude.



FIGURE 3.5: Missile Datcom Flowchart

3.7 Simulink Model

Simulink is a MATLAB-based graphical programming environment for modeling, simulating and analyzing multi-domain dynamical systems. A multi-domain modeling and simulation environment for engineers and scientists who design controls, wireless, and other dynamic systems. Simulink is the platform for Model-Based Design that supports system-level design, simulation, automatic code generation, and continuous test and verification of embedded systems.

The Figure 3.6 below shows the Matlab Simulink diagram that are made to create the simulation for this thesis. From the top left are the blockset of thrust force, the database given from Lapan is used to simulate the drag force which a 16 seconds thrust averaging at 95 000 N. Second left blockset are the gravity force that use the wgs84 blockset. Third blockset are the aerodynamic forces and the forth blockset are the rocket dynamics which feed inertia part. 6DOF (Euler Angles) are used to calculate the equations of motions of the rocket which will then generate overall flight data.

3.7.1 Simulink Blockset

The thrust blockset shown in Figure 3.7 will configure the thrust force for the rocket. A look-up table the will be outputting a 17 seconds thrust at an average 95 kN, the thrust will be simultaneously reduced during the flight by simulated drag force that are obtained from the atmosphere pressure.

The gravity blockset shown in Figure 3.8 will calculated the gravity force that are experienced by the rocket, the forces are adjusted to the rocket to earth distances and rocket orientation. Significant gravity changes will happened during the early phase of the flight, which at the time the rocket fuel are burnt out and reducing the total rocket weight to less than a half.

The aerodynamic blockset shown in Figure 3.9 will calculated the aerodynamic forces applied to rocket. the aerodynamic coefficients database that are used for the calculation is obtained from the Missile-Datcom simulation which created from a predicted flight situation that the rocket will endure in terms of its altitude, attitude, and velocity.



FIGURE 3.6: RX450 Simulink (Top Model)

The mass and inertia blockset shown in Figure 3.10 will configure the rocket mass, center of gravity position (X-cg), inertia tensor matrix and rate of change of the inertia tensor matrix.

The Coriolis blockset applied only during the thrust is applied. Figure 3.11 are the coriolis blockset on the simulink.

3.7.2 Simulations

The simulation will imitate a rx450 sub-orbital experimental flight, the flight is launch from Lapan Pameungpeuk launch site at the south coastline of Garut. the experimental flight regularly done for at least twice a year. The Rocket 6 m tall



FIGURE 3.7: Simulink Thrust



FIGURE 3.8: Simulink Gravity

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FIGURE 3.9: Simulink Aerodynamic



FIGURE 3.10: Simulink Mass Inertia



FIGURE 3.11: Simulink Coriolis

rocket will burn the $95 \,\mathrm{kN}$ thrust for about 16 seconds at 60^{circ} azimuth towards the south pole and from several test that has been done before the rocket has able to reach 84 km in orbital altitude.



FIGURE 3.12: RX450 Thrust Profile

CHAPTER 4 RESULTS AND DISCUSSIONS

4.1 Simulation Parameter

Simulation	Parameter
Solver	ode45
Time-Step	Fixed
Step size	0.001
Initial Step size	auto
Relative tolerance	$1e^{-3}$
Absolute tolerance	auto
Terminal condition	$-Z_I$
Max-time	350 sec

TABLE 4.1: Simulation Parameter

Shown in Table 4.1 Ode45 is based on an explicit Runge-Kutta (4,5) formula The MATLAB command ode45 performs a direct numerical integration of a set of differential equations from to some final time tf, The "45" part specifies the type of numerical integration used. where t-span = $[t_0 - tf]$, integrates the system of differential equations y' = f(t, y) from t_0 to t_F with initial conditions y_0 . Each row in the solution array y corresponds to a value returned in column vector t.

4.2 Simulation Result for Free Motion Flight

The free flight simulation were made to create a neutral environment for the rocket flight and to setup a standard trajectory that will then set as a reference trajectory to evaluate the control system performance after being given the rocket is given a disturbances that will deviate the reference trajectory.

NUMERICAL ANALYSIS OF THE RX450 FREE-MOTION FLIGHT

Timeline (sec)	Event
T+0.0	Simulation Start
T + 0.0	Thrust Force Applied
T+16.0	Main Engine Off and Vz Gradually decrease
T+17.0	Alpha and Pitch movement
T + 45.0	Vx stop decrease, Air Pressure (Pa) closing to 0
T + 90.0	Alpha and Pitch angle drift
T+150.0	Rocket reached apogee
T+200.0	Rocket start to tumbling down and oscilate
T+275.0	Air Pressure increase
T + 306.0	Rocket $Alt = 0$

 TABLE 4.2: Event Timeline

4.2.1 Altitude and Range

First Figure 4.1 shows the Apogee which is the highest altitude reach during the free flight simulation test is around 92 km. this can be consider to be similar to what Lapan has stated on their website for the apogee reached by the RX450 (Agustina, 2020). The overall distance covered by the rocket is shown in Figure 4.1, for the horizontal displacement xi the rocket reach 115 km. The 3 Dimension trajectory below also show that there is no lateral displacement during this flight.

4.2.2 Attitude

During the flight the rocket is relatively stable, this was shown by the pitch angle in Figure 4.2 is relatively stable, although 3 significant movement is happened during the flight. the first move during the first few seconds of the flight which startled the rocket with the thrust force. second oscillation happened right after the main engine shut-off, this movement may happened due to a sudden decrease of thrust force which can be seen from Figure 4.2. which shows the reduction of the vertical velocity during that exact time and also the aerodynamic force is significantly dominate the total force acting on the rocket. third oscillation happened after the aircraft beginning to descent, this may happened due to the weight at the fore section of the rocket is significantly bigger after than the aft after the solid fuel being emptied.



FIGURE 4.1: Geopotential Altitude (Height) v Horizontal Displacement (xi) and Flight Time (s)

For the Roll and Yaw movement that are shown in Figure 4.3 during the Undisturbed flight is relatively stable. The rocket did not show any movement during several critical phase at T+17 seconds during the main engine off and T+277 seconds which the rocket start to hit the thicker part of the atmosphere and the air pressure increases significantly during. The figure below shows that the Roll an Yaw movement of the rocket stays at 0 thru out the flight.

4.2.3 Velocity

The overall velocity of the rocket in Figure 4.4 shows that it will reach a mach 4.7 Ma as it max speed which occur right at the main engine shutoff. The Vertical velocity as it shown in Figure 4.4 will decrease up to the apogee height, in which right after that it will start to accelerate back towards the earth and gradually

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FIGURE 4.2: Pitch Angle and Alpha

decreasing the acceleration rate after the rocket reach the lower part of the atmosphere which can be seen started at 40 km of altitude. The horizontal velocity shows a rather different profile, it was start decreasing right after the main engine shutdown but right after the rocket get significantly higher in which the drag force is becoming lower the horizontal velocity remains constant as to what happened when there is no significant drag force.

4.2.4 Solver and Step Size Comparison

To further prove that the result of this simulation are consistent and accurate the simulation are also tested with different sets of parameter. The set of parameter that are changed are the solver and step-size, between the two parameter we run the unperturbed simulation to see if there are significant differences between them.



FIGURE 4.3: Roll and Yaw

Due to the limitation of the computing power, the smallest step-size that are tested for the unperturbed flight is 0.0002 sec rather than 0.0001 sec. For the comparison of the higher step size, the number that will be use is 0.001 sec and 0.01 sec. For the solver, this test will run 4 different set of solver which include ODE8 (Dormand-Prince), ODE5 (Dormand-Prince), ODE4 (Runge-Kutta), ODE3 (Bogacki-Shampine). All 4 of the solver will be tested with the step size of 0.001 and 0.01.

Figure 4.5 are the comparison of the range or horizontal displacement between step size 0.0002, 0.001 and 0.01. From the result the rocket are all reaching the same altitude and horizontal displacement, so it can be concluded that there are no significant differences between all 3 sets of simulation.

Figure 4.6 shows the attitude of the rocket in Alpha and pitch number for the test with step size 0.0002. Figure 4.7 and Figure 4.8 shows the comparison between



FIGURE 4.4: Vertical Velocity(vz) and Horizontal Velocity (vx)

the 4 different solver that are tested with the step size of 0.001 and 0.01. From all of the 3 figures mentioned above, it can be concluded that the aircraft motion are relatively the same between all different sets of parameter that are used.

4.3 Atmospheric Influence

The air pressure acting as one of the aerodynamic drag on to the rocket thus it will effect mainly on to the horizontal velocity, since gravity force have the better effect towards the vertical velocity. The Figure 4.4 shows that the air pressure only come to a greater effect right before the height of 40 Km, above that the air pressure are to minuscule and become insignificant. Thus why from Figure 4.4 the Horizontal velocity remain constant after 45 seconds of flight and then started to



FIGURE 4.5: Trajectory Comparison with Step Size 0.0002 (Top), 0.001(Middle), 0.01(Bottom)



FIGURE 4.6: Alpha and Pitch angle with Parameter Step size : 0.0002, ODE4 (Runge-Kutta)

become effective again after 277 sec in which the rocket has entered the thicket part of the atmosphere, this can be seen from the mach number Figure 4.9.

4.4 Simulation Result for Perturbation Influenced Flight

To simulate a turbulences that occur during the flight, there are additional blockset before the input of aerodynamic force and moment that combined with the Vb input. The Vb Body velocity contain 3 vectors value, each of those vector value will then be added reduce or added to simulate the aerodynamic forces and the disturbances from the local atmosphere.

The Turbulence setup is step signal blockset that will added scalar vector value towards the input of aerodynamic subsystem, The Body Velocity (Vb) that apply as an input will be enumerate with the step signal blockset. The result will be a vector value added directly towards the 3 directional vector of the rocket.



FIGURE 4.7: Pitch and Alpha Comparison Step size '0.001' ODE3 (Top Left), ODE4 (Top Right), ODE5 (Bottom Left), ODE8 (Bottom Right)



FIGURE 4.8: Pitch and Alpha Comparison Step size '0.01' ODE3 (Top Left), ODE4 (Top Right), ODE5 (Bottom Left), ODE8 (Bottom Right)



FIGURE 4.9: Flight Time v Mach Number

4.4.1 Altitude and Range

First Figure 4.10 shows the apogee or the highest altitude of the perturbed flight is 59 km Km, this is significantly less compared to the free motion flight that reached 92 km. For the horizontal displacement xi for the perturbed flight reach 98 km while the undisturbed flight reached 115 km. The 3 Dimension trajectory below (right side of Figure 4.10) also show that there are lateral displacement during this perturbation flight at about 37 km.

4.4.2 Attitude

For the perturbed flight there are significant difference on the rocket motion. Shown in Figure 4.11 After being applied with the small perturbation the rocket start to roll, this rolling movement did not stop until the rocket reached the ground. The rocket also starts to Yaw although it dampened it self immediately but not into 0 yaw angle. The Yaw movement starts to oscillate again during the re-entry part of the flight, when the rocket start to hit the thicker part of the atmosphere.

Timeline (sec)	Event
T + 0.0	Simulation Start
T+0.0	Thrust Force Applied
T+16.0	Main Engine Off and Vz Gradually decrease
T+18.0	Perturbation Applied
T+19.0	Rocket start to Roll and Yaw
T + 40.0	Vx stop decrease, Air Pressure (Pa) closing to 0
T + 90.0	Alpha and Pitch angle drift
T+120.0	Rocket reached apogee
T+220.0	Air Pressure increase
T+235.0	Rocket $Alt = 0$

TABLE 4.3: Event Timeline

The pitch however shown in Figure 4.12 have a similar with the free motion flight, the alpha number shows that the rockets able to dampened the oscillation.

4.4.3 Velocity

The velocity of the rocket shows that it also reach a mach 4.7 Ma similar to the free motion flight, this is because the perturbation applied after the main engine is off. The rocket pretty much show the exact same velocity profile with respect to the free motion flight, where the vertical velocity as it shown in Figure 4.13 will decrease up to the apogee height, in which right after that it will start to accelerate back towards the earth and gradually decreasing the acceleration rate after the rocket reach the lower part of the atmosphere which can be seen started at 40 Km of altitude.

The horizontal velocity of the disturbed flight shown in Figure 4.13 similar profile with the undisturbed flight, it was start decreasing right after the main engine shutdown but right after the rocket get significantly higher in which the drag force is becoming lower the horizontal velocity remains constant as to what happened when there is no significant drag force.

Lateral Velocity show a different profile compared to the free motion flight. the Lateral velocity starts to obtained some speed right after it has been applied with lateral perturbation, after that the rocket veering of with a constant speed similar



FIGURE 4.10: Geopotential Altitude (Height) v Horizontal Displacement (xi) and Flight Time (s)

to the horizontal velocity due to significantly low air pressure and then starts to reduce significantly when it reached the thicker part of the atmosphere.

4.5 Trajectory Comparison Analysis

In this section the free-flight simulation will be compared with the disturbancegiven trajectory. The comparison will consist of several figures of graph that shows the range (Horizontal Displacement), altitude (Vertical Displacement), pitch, velocity, flight time. The perturbation forces will consist of vector value acting on the Vbx and Vbz, the disturbance force acting to the opposite direction of the vehicle body axis which equal to total force of (-)Vbx and (-)Vbz.



FIGURE 4.11: Roll v Yaw

The first comparison is the Altitude (height) and the Horizontal Displacement (range) shown in Figure 4.14. The result show that the disturbed flight trajectory have reached a lower highest point of altitude (apogee) and shorter distance cover on the horizontal displacement (range).

The second comparison shown in Figure 4.15 is the horizontal velocity of the rocket, the highest velocity achieved by both of the situation is exactly the because the disturbance forces are applied 1 second right after the main engine off and only during the predicted atmospheric flight at around below 40 km of altitude.

For the vertical velocity shown in Figure 4.16 however did not show a significant differences between both scenario, the Perturbation forces helped decrasing the vertical velocity for a several seconds after the main engine off. This is because the disturbance force acting to the opposite towards the direction of the flight.

The forth comparison shows the Pitch angle and the Alpha number of the



FIGURE 4.12: Pitch and Alpha



FIGURE 4.13: Vertical Velocity(vz) and Horizontal Velocity (vx)

rocket. From the Figure 4.17 4.18 below, the result shows that both flight scenario had pretty much the same attitude for the entirety of the flight. Starting from the Launch which both graph shows that there is a minor movement after the rocket leaving the launch pad, the second movement happened after the main engine off, and the last one is the biggest pitch number that occur during the rocket is significantly out of the atmospheric pressure.

The roll and yaw movement is only occurs during the pertubed flight shown in Figure 4.19. both movement starts to initiate only right after the perturbation is applied.

Th Lateral displacement also only occurs during the perturbed flight. shown in Figure 4.20 due to the perturbation right after the main engine shutdown.



FIGURE 4.14: Free Flight v Disturbed Flight

Event	Free Flight	Disturb Flight
Apogee (Highest Altitude)	$92\mathrm{km}$	$59\mathrm{km}$
Horizontal Displacement	$115\mathrm{km}$	$98\mathrm{km}$
Lateral Displacement	$0\mathrm{km}$	$35\mathrm{km}$
Maximum Velocity	$4.7 {\rm Ma}$	4.7 Ma
Lateral Velocity (Avg)	$0\mathrm{m/s}$	$149\mathrm{m/s}$
Vertical Velocity (up to Apogee)	$613\mathrm{m/s}$	$500\mathrm{m/s}$
Horizontal Velocity (Avg)	$376\mathrm{m/s}$	$417\mathrm{m/s}$
Flight Time	$306\mathrm{s}$	$235\mathrm{s}$

 TABLE 4.4: Event Timeline



FIGURE 4.15: Horizontal Velocity v Flight Time



FIGURE 4.16: Vertical Velocity v Flight Time



FIGURE 4.17: Attitude (Alpha) v Flight Time



FIGURE 4.18: Attitude (Pitch) v Flight Time



FIGURE 4.19: Attitude (Roll and Yaw) v Flight Time



FIGURE 4.20: Geopotential Altitude (Height) v Horizontal Displacement (xi)
CHAPTER 5 SUMMARY, CONCLUSION, RECOMMENDATION

5.1 Summary

To summarize this thesis, the Simulink blocksets were constructed based on the derives equations. These equations were then built into the Simulink blocks which later then connected to build the full numerical simulation environment. The top model of the simulation are shown in Figure 3.6.

The free-motion flight of the RX-450 rocket showed that the motion of the rocket was overall relatively stable during all the phases of the flight. From the free-motion flight, one can conclude that there were three significant phases that have potential to destabilize the rocket: launch, engine ehut-off, and atmospheric re-entry. During all those three phases, the rocket was capable to return and dampened out back into a stable position. But from Figure 4.2, the highest α reached by the rocket was during the high-atmospheric flight where it went up to 55°. During the descend phase the rocket managed to dampened the alpha and the oscillation and eventually stopped after the rocket are naturally stable within this part of the simulation.

From the perturbed flight, the lateral perturbation sent the aircraft into a yaw and roll movements. The initial perturbation significantly deviated the aircraft from the original heading. The yaw angle of the rocket achieved 90° before it settled back although never reached zero yaw angle as can be seen on Figure 4.19. The Roll angle of the rocket always stayed at the same rolling rate for almost the entire flight as seen also in Figure 4.19. The pitch angle was relatively constant although showed small oscillation — due to the Lateral perturbation — although the values was negligible compared to the free-motion flight.

The baseline performance for the free motion flight achieved bigger horizontal and vertical distances but no lateral displacement was produced. While the perturbed flight only was able to reach 58 km of maximum vertical distance, the apogee for for the free flight case was 98 km. The big different lies in the lateral distance it cover at about 36 km. Compared to (Agustina, 2020) the obtained performance of the simulated flight were considered to be realistic enough to represent the real flight performance.

5.2 Conclusions

This thesis have managed to build a numerical simulation for the RX-450 rocket investigated two different types of flight condition. And here are the conclusions obtained from this thesis:

- From both scenarios we could observe that with or without perturbation the rocket could still fly relatively stable (under small perturbation assumption);
- For the free-motion flight, RX-450 exhibited motion that was overall relatively stable during all the phases of the flight;
- When undisturbed, the rocket experienced an oscillation it pitched up significantly from the initial flight direction — which later dampened naturally at lower altitude.
- When small perturbation applied, the perturbation caused the rocket to roll while descending;
- The apogee reached by RX-450 during the test flight from the undisturbed simulations was 92 km which fell within similar values given in the official report by LAPAN.

5.3 Recommendation

For further, works that are recommended as extensions of this thesis are:

- To investigate the effects of small and large perturbation for non-nominal thrust profile;
- To introduce a control system for the rocket by using the four of the movable fins located at the aft;
- To add another sets of control system that is capable to control the rocket outside of the atmosphere such as RCS (Reaction Control System).

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Appendices

Appendix A: Python Codes

```
#!/usr/bin/env python3
import numpy as np
from izzo_lib import lambert_izzo, lambert_univ
from jgm2_constants import MIU N, ER, VU, TU
from keplerian_elements import randv, elorb
from maneuvers import hohman_trans
from numpy.linalg import norm
import matplotlib.pyplot as plt
def semi_p(a, e):
   res = a * (1 - e * 2)
   return res
# Initial orbit
e0 = 0
h0 = 220 # altitude (km)
a0 = (ER + h0) / ER
p0 = semi_p(a0, e0)
inc0 = 0
OmegaO = O
omega0 = 0
nu0 = 0
coe0 = [p0, e0, inc0, Omega0, omega0, nu0]
r0, v0 = randv(coe0)
```

```
# Final orbit
e1 = 0
h1 = 35786 # altitude (km)
a1 = (ER + h1) / ER
p1 = semi_p(a1, e1)
inc1 = 0
Omega1 = 0
omega1 = 0
nu1 = 180
coe1 = [p1, e1, inc1, Omega1, omega1, nu1]
r1, v1 = randv(coe1)
r0_{-} = norm(r0)
r1_ = norm(r1)
a trans, tau trans, Dva, Dvb = hohman trans(r0, r1)
tofs = np.linspace(0.7 * tau_trans, 10 * tau_trans, 500)
Dvs = np.array([])
for tof in tofs:
    ((v0_lam, v1_lam),) = lambert_izzo(r0, r1, tof=tof, M=0,)
    # v0 lam, v1 lam = lambert univ(r0, r1, tof=tof, short=True,)
   Dv0 = norm(v0_lam - v0)
   Dv1 = norm(v1_lam - v1)
   Dv = Dv0 + Dv1
   Dvs = np.append(Dvs, Dv)
fig, ax = plt.subplots(figsize=(9, 9))
ax.plot(tofs * TU / 3600, Dvs * VU, "o")
```

```
ax.set_xlabel("tof (hour)")
ax.set_ylabel("DV (km/s)")
plt.grid("both")
plt.show()
```

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